CORRECTION DU CB N°1

1) $Z = \dfrac{\left(-1+\sqrt{3}i\right)^6}{\left(2+2i\right)^4} = \dfrac{\left(2e^{i\frac{2\pi}{3}}\right)^6}{\left(2\sqrt{2}e^{i\frac{\pi}{4}}\right)^4} = \dfrac{2^6 e^{2i\pi}}{2^6 e^{i\pi}} = -1 = \cos(\pi) + i\sin(\pi)$

2) S = {1 + i ; -1 + i ; -1 – i ; 1 – i} $= \left\{ \sqrt{2} e^{i\left(\frac{\pi}{4}+k\frac{\pi}{2}\right)} \;/\; k \in \{0,1,2,3\} \right\}$

3) A = $\cos^3(x)\sin^2(x) = -\dfrac{1}{16}(\cos(5x) + \cos(3x) - 2\cos(x))$

4) $e^{i\theta} + e^{i\theta'} = e^{i\frac{\theta+\theta'}{2}} \times 2\cos\left(\dfrac{\theta-\theta'}{2}\right)$.

Si $\theta - \theta' = (2k+1)\pi \;\; (k \in \mathbb{Z})$, $e^{i\theta} + e^{i\theta'} = 0$.

Sinon : $e^{i\theta} + e^{i\theta'} = e^{i\left(\frac{\theta}{2}+\frac{\theta'}{2}\right)}\left(e^{i\left(\frac{\theta}{2}-\frac{\theta'}{2}\right)} + e^{-i\left(\frac{\theta}{2}-\frac{\theta'}{2}\right)}\right) = 2e^{i\left(\frac{\theta}{2}+\frac{\theta'}{2}\right)}\cos\left(\dfrac{\theta}{2}-\dfrac{\theta'}{2}\right)$

$\left|e^{i\theta} + e^{i\theta'}\right| = 2\left|\cos\left(\dfrac{\theta-\theta'}{2}\right)\right|$ et $\arg\left(e^{i\theta} + e^{i\theta'}\right) = \begin{cases} \dfrac{\theta+\theta'}{2}[2\pi] & \text{si } \cos\left(\dfrac{\theta-\theta'}{2}\right) > 0 \\ \dfrac{\theta+\theta'}{2} + \pi \; [2\pi] & \text{si } \cos\left(\dfrac{\theta-\theta'}{2}\right) < 0 \end{cases}$

CORRECTION DU CB N°1

1) $Z = \dfrac{\left(1+\sqrt{3}i\right)^6}{\left(2-2i\right)^4} = \dfrac{\left(2e^{i\frac{\pi}{3}}\right)^6}{\left(2\sqrt{2}e^{i\frac{3\pi}{4}}\right)^4} = \dfrac{2^6 e^{2i\pi}}{2^6 e^{i\pi}} = -1 = \cos(\pi) + i\sin(\pi)$

2)

$S = \left\{\sqrt{3}e^{i\left(\frac{\pi}{4}+k\frac{\pi}{2}\right)} \,/\, k \in \{0,1,2,3\}\right\} = \left\{\sqrt{\frac{3}{2}}(1 + i) \; ; \; \sqrt{\frac{3}{2}}(-1 + i) \; ; \; \sqrt{\frac{3}{2}}(-1 - i) \; ; \; \sqrt{\frac{3}{2}}(1 - i)\right\}$

3) $A = \cos(x)\sin^4(x) = \frac{1}{8}\cos(x) - \frac{3}{16}\cos(3x) + \frac{1}{16}\cos(5x)$

4) $e^{i\theta} - e^{i\theta'} = e^{i\frac{\theta+\theta'}{2}} \times 2i\sin\left(\frac{\theta-\theta'}{2}\right)$.

Si $\theta - \theta' = 2k\pi \;\; (k \in \mathbb{Z})$, $e^{i\theta} - e^{i\theta'} = 0$.

Sinon : $e^{i\theta} - e^{i\theta'} = e^{i\left(\frac{\theta}{2}+\frac{\theta'}{2}\right)}\left(e^{i\left(\frac{\theta}{2}-\frac{\theta'}{2}\right)} - e^{-i\left(\frac{\theta}{2}-\frac{\theta'}{2}\right)}\right) = 2ie^{i\left(\frac{\theta}{2}+\frac{\theta'}{2}\right)}\sin\left(\frac{\theta}{2}-\frac{\theta'}{2}\right)$

$\left|e^{i\theta} - e^{i\theta'}\right| = 2\left|\sin\left(\frac{\theta-\theta'}{2}\right)\right|$ et $\arg\left(e^{i\theta} - e^{i\theta'}\right) = \begin{cases} \dfrac{\theta+\theta'+\pi}{2}[2\pi] & \text{si } \sin\left(\dfrac{\theta-\theta'}{2}\right) > 0 \\ \dfrac{\theta+\theta'-\pi}{2}[2\pi] & \text{si } \sin\left(\dfrac{\theta-\theta'}{2}\right) < 0 \end{cases}$