CORRECTION DU CB N°10

i) $\quad I_1=\int_5^7 \frac{x-3}{x^2-5x+4}dx$

$$=\int_5^7\left(\frac{1}{3(x-4)}+\frac{2}{3(x-1)}\right)dx=\left[\frac{1}{3}\ln(x-4)+\frac{2}{3}\ln(x-1)\right]_5^7=\frac{-2}{3}\ln(2)+\ln(3)$$

ii) $\quad I_2=\int_1^2 \frac{x^3}{\sqrt{2x^2+1}}dx$

$$=\left[\frac{x^2}{2}\sqrt{2x^2+1}\right]_1^2-\int_1^2\left(x\sqrt{2x^2+1}\right)dx=\left(6-\frac{\sqrt{3}}{2}\right)-\left[\frac{1}{6}(2x^2+1)^{\frac{3}{2}}\right]_1^2=\frac{3}{2}$$

iii) $\quad I_3=\int_0^{\frac{\pi}{2}}\cos(x)\sin(3x)dx$

$$=\int_0^{\frac{\pi}{2}}\left(4\cos^3(x)\sin(x)-\cos(x)\sin(x)\right)dx=\left[-\cos^4(x)+\frac{1}{2}\cos^2(x)\right]_0^{\frac{\pi}{2}}=\frac{1}{2}$$

iv) $\quad I_4=\int_0^1\frac{2x}{(3x^2+1)^2}dx \quad =\left[\frac{-1}{3(3x^2+1)}\right]_0^1=\frac{1}{4}$

v) $\quad I_5=\int_0^1\sqrt{1-x^2}dx$

$$=\int_0^{\frac{\pi}{2}}\left(\sqrt{1-\sin^2(t)}\cos(t)\right)dt=\int_0^{\frac{\pi}{2}}\cos^2(t)dt=\int_0^{\frac{\pi}{2}}\frac{1+\cos(2t)}{2}dt=\left[\frac{t}{2}+\frac{1}{4}\sin(2t)\right]_0^{\frac{\pi}{2}}=\frac{\pi}{4}$$

vi) $\quad I_6=\int_0^{\ln(2)}\frac{1}{\sqrt{2e^x-1}}dx \quad =\int_1^{\sqrt{3}}\frac{1}{u}\times\frac{2u}{1+u^2}du=\left[2\operatorname{Arc}\tan(u)\right]_0^{\sqrt{3}}=\frac{\pi}{6}$

vii) $\quad I_7=\int_0^{\frac{\pi}{8}}\frac{1}{\cos^2(2x)}dx \quad =\left[\frac{1}{2}\tan(2x)\right]_0^{\frac{\pi}{8}}=\frac{1}{2}$

CORRECTION DU CB N°10

i) $\mathrm{I}_1 = \int_5^6 \frac{x-5}{x^2-5x+4}\,dx$

$= \int_5^6 \left(\frac{-1}{3(x-4)} + \frac{4}{3(x-1)} \right) dx = \left[-\frac{1}{3}\ln(x-4) + \frac{4}{3}\ln(x-1) \right]_5^6 = -3\ln(2) + \frac{4}{3}\ln(5)$

ii) $\mathrm{I}_2 = \int_0^1 \frac{2x^3}{\sqrt{x^2+1}}\,dx$

$= \left[2x^2\sqrt{x^2+1} \right]_0^1 - \int_0^1 \left(4x\sqrt{x^2+1} \right) dx = 2\sqrt{2} - \left[\frac{4}{3}\left(x^2+1\right)^{\frac{3}{2}} \right]_0^1 = \frac{-2}{3}\sqrt{2} + \frac{4}{3}$

iii) $\mathrm{I}_3 = \int_{\frac{\pi}{2}}^{\pi} \cos(3x)\sin(x)\,dx$

$= \int_{\frac{\pi}{2}}^{\pi} \left(2\cos^3(x)\sin(x) - \cos(x)\sin(x) - 2\sin^3(x)\cos(x) \right) dx$

$= \left[-\frac{1}{2}\cos^4(x) + \frac{1}{2}\cos^2(x) - \frac{1}{2}\sin^4(x) \right]_{\frac{\pi}{2}}^{\pi} = \frac{1}{2}$

iv) $\mathrm{I}_4 = \int_1^2 \frac{3x}{\left(2x^2+1\right)^2}\,dx = \left[\frac{-3}{4\left(2x^2+1\right)} \right]_1^2 = \frac{1}{6}$

v) $\mathrm{I}_5 = \int_0^1 \sqrt{x^2+1}\,dx$

$= \int_0^{\ln\left(1+\sqrt{2}\right)} \left(\sqrt{1+\mathrm{sh}^2(t)}\,\mathrm{ch}(t) \right) dt = \int_0^{\ln\left(1+\sqrt{2}\right)} \mathrm{ch}^2(t)\,dt = \int_0^{\ln\left(1+\sqrt{2}\right)} \frac{1+\mathrm{ch}(2t)}{2}\,dt$

$= \left[\frac{t}{2} + \frac{1}{4}\mathrm{sh}(2t) \right]_0^{\ln\left(1+\sqrt{2}\right)} = \frac{1}{2}\sqrt{2} + \frac{1}{2}\ln\left(1+\sqrt{2}\right)$

vi) $\mathrm{I}_6 = \int_{\ln(1/3)}^{\ln(2/3)} \frac{1}{\sqrt{3e^x-1}}\,dx = \int_0^1 \frac{1}{u} \times \frac{2u}{1+u^2}\,du = \left[2\mathrm{Arc}\tan(u) \right]_0^1 = \frac{\pi}{2}$

vii) $\mathrm{I}_7 = \int_0^{\frac{\pi}{12}} \frac{1}{\cos^2(3x)}\,dx = \left[\frac{1}{3}\tan(3x) \right]_0^{\frac{\pi}{12}} = \frac{1}{3}$