C.B. N° 9	Correction	2012-2013

Calculer les intégrales suivantes :

i) $$I_1=\int_2^3 \frac{3x^4-3x^3-2x^2+1}{x^2\left(x-1\right)^2}dx =\int_2^3\left(3+\frac{2}{x}+\frac{1}{x^2}+\frac{1}{x-1}-\frac{1}{(x-1)^2}\right)dx=\frac{8}{3}+2\ln(3)-\ln(2)$$

ii) $$I_2=\int_0^{\frac{\sqrt{2}}{2}}\frac{1}{\left(3x^2+1\right)\left(2x^2+1\right)}dx=\int_0^{\frac{\sqrt{2}}{2}}\left(\frac{3}{3x^2+1}-\frac{2}{2x^2+1}\right)dx$$

$$=\left[\sqrt{3}\text{Arc}\tan\left(\sqrt{3}x\right)-\sqrt{2}\text{Arc}\tan\left(\sqrt{2}x\right)\right]_0^{\frac{\sqrt{2}}{2}}=\sqrt{3}\text{Arctan}\left(\frac{\sqrt{6}}{2}\right)-\sqrt{2}\frac{\pi}{4}$$

iii) $$I_3=\int_1^2\frac{2x^2-1}{x\left(x^2-x+1\right)}dx=\int_1^2\left(\frac{-1}{x}+\frac{3x-1}{x^2-x+1}\right)dx$$

$$=\left[\ln(x)+\frac{3}{2}\ln\left(x^2-x+1\right)+\frac{\sqrt{3}}{3}\text{Arc}\tan\left(\frac{2}{\sqrt{3}}\left(x-\frac{1}{2}\right)\right)\right]_1^2=-\ln(2)+\frac{3}{2}\ln(3)+\frac{\pi\sqrt{3}}{18}$$

iv) $$I_4=\int_1^2\frac{x^3}{\sqrt{3x^2+1}}dx=\left[x^2\times\frac{1}{3}\sqrt{3x^2+1}\right]_1^2-\int_1^2\frac{2}{3}x\sqrt{3x^2+1}dx$$

$$=\left[x^2\times\frac{1}{3}\sqrt{3x^2+1}\right]_1^2-\left[\frac{2}{27}\left(3x^2+1\right)^{\frac{3}{2}}\right]_1^2=\frac{10}{27}\sqrt{13}-\frac{2}{27}$$

v) $$I_5=\int_0^{\frac{\pi}{2}}\cos^3(x)\sin^4(x)dx=\int_0^{\frac{\pi}{2}}\left(\sin^4(x)-\sin^6(x)\right)\cos(x)dx$$

$$=\left[\frac{1}{5}\sin^5(x)-\frac{1}{7}\sin^7(x)\right]_0^{\frac{\pi}{2}}=\frac{2}{35}$$

vi) $$I_6=\int_{\ln\left(\frac{1}{5}\right)}^{\ln\left(\frac{2}{5}\right)}\frac{1}{\sqrt{5e^x-1}}dx=\int_0^1\frac{1}{t}\times\frac{2t}{1+t^2}dt=\left[2\text{Arc}\tan(t)\right]_0^1=\frac{\pi}{2}$$

C.B. N° 9	Correction	2012-2013

Calculer les intégrales suivantes :

i) $$I_1=\int_2^3 \frac{4x^4-7x^3+6x^2-3x+1}{x^2(x-1)^2}dx=\int_2^3\left(4-\frac{1}{x}+\frac{1}{x^2}+\frac{2}{x-1}+\frac{1}{(x-1)^2}\right)dx$$

$$=3\ln(2)-\ln(3)+\frac{14}{3}$$

ii) $$I_2=\int_0^{\frac{\sqrt{3}}{3}}\frac{11x^2+4}{(3x^2+1)(2x^2+1)}dx=\int_0^{\frac{\sqrt{3}}{3}}\left(\frac{1}{3x^2+1}+\frac{3}{2x^2+1}\right)dx$$

$$=\left[\frac{\sqrt{3}}{3}\operatorname{Arc\,tan}\left(\sqrt{3}x\right)+\frac{3\sqrt{2}}{2}\operatorname{Arc\,tan}\left(\sqrt{2}x\right)\right]_0^{\frac{\sqrt{3}}{3}}=\sqrt{3}\frac{\pi}{12}+\frac{3\sqrt{2}}{2}\operatorname{Arctan}\left(\frac{\sqrt{6}}{3}\right)$$

iii) $$I_3=\int_1^2\frac{x^2-x+3}{x(x^2-x+1)}dx=\int_1^2\left(\frac{3}{x}+\frac{-2x+2}{x^2-x+1}\right)dx$$

$$=\left[3\ln(x)-\ln\left(x^2-x+1\right)+\frac{2\sqrt{3}}{3}\operatorname{Arc\,tan}\left(\frac{2}{\sqrt{3}}\left(x-\frac{1}{2}\right)\right)\right]_1^2=3\ln(2)-\ln(3)+\frac{\pi\sqrt{3}}{9}$$

iv) $$I_4=\int_1^2\frac{2x^3}{\sqrt{4x^2+1}}dx=\left[x^2\times\frac{1}{2}\sqrt{4x^2+1}\right]_1^2-\int_1^2 x\sqrt{4x^2+1}dx$$

$$=\left[x^2\times\frac{1}{2}\sqrt{4x^2+1}\right]_1^2-\left[\frac{1}{12}(4x^2+1)^{\frac{3}{2}}\right]_1^2=\frac{7\sqrt{17}-\sqrt{5}}{12}$$

v) $$I_5=\int_0^{\frac{\pi}{2}}\cos^4(x)\sin^3(x)dx=\int_0^{\frac{\pi}{2}}\left(\cos^4(x)-\cos^6(x)\right)\sin(x)dx$$

$$=\left[\frac{-1}{5}\cos^5(x)+\frac{1}{7}\cos^7(x)\right]_0^{\frac{\pi}{2}}=\frac{2}{35}$$

vi) $$I_6=\int_{\ln\left(\frac{1}{7}\right)}^{\ln\left(\frac{2}{7}\right)}\frac{1}{\sqrt{7e^x-1}}dx=\int_0^1\frac{1}{t}\times\frac{2t}{1+t^2}dt=\left[2\operatorname{Arc\,tan}(t)\right]_0^1=\frac{\pi}{2}$$